Automatic Control (1)

By

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كلية الهندسة بشبرا

FACULTY OF ENGINEERING AT SHOUBRA

Lecture (4)

Course Title: **Automatic Control (1)**

Course Code: **ELE 314**

Contact Hours: **4.**

= [2 Lect. + 2 Tut + 0 Lab]

Assessment:

Final Exam: 60%.

Midterm: 20%.

Year Work & Quizzes: 20%.

Experimental/Oral: 20%.

Textbook:

1- Benjamin C. Kuo " Automatic control systems" 9th ed., John Wiley & Sons, Inc. 2010.

2- Katsuhiko Ogata, "Modern Control Engineering", 4th Edition, 2001.

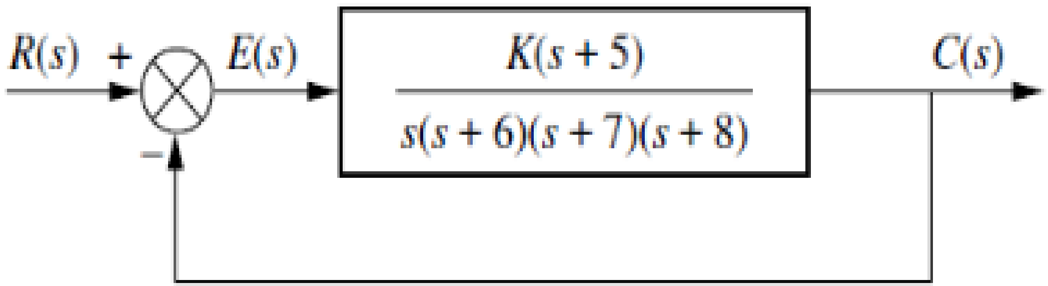
Course Description

- State space variables, Solving state space equation, Basic definitions in modern control (Observability and Controllability), Transfer function analysis, Error analysis, Static and dynamic error coefficients, Steady state error, Error characteristics, Basic control action and industrial automatic control (P, PI, PID controllers), Transient response for control systems of first and second order, Poles / Zeros, Eigen value and stability of multivariable system, Stability analysis, Routh-Hurwitz criterion.

Control Design using Steady State error

Exercise (1)

For the system shown below, find K so that there is 10% error in the steady state



Since the system type is 1, the error stated in the problem must apply to a ***ramp input***; only a ramp yields a finite error for in a type 1 system. Thus,

$$K_v = \lim_{S \to 0} S\,G(S) = \frac{K \times 5}{6 \times 7 \times 8} = \frac{5K}{336}$$

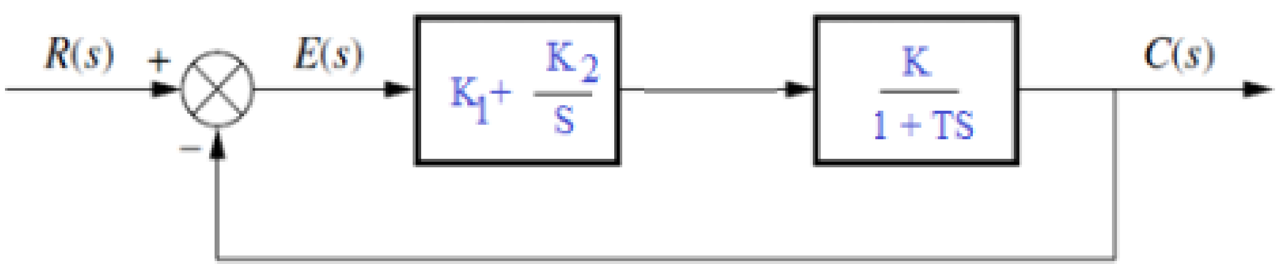
$$E_{ss} = \frac{336}{5K} = 0.1$$

$$K = 672$$

Exercise (2)

An engine speed control system is shown below.

1) Calculate E_{ss} for step input with magnitude A when $K_2 = 0$
2) Calculate E_{ss} for step input with magnitude A when $K_2 \neq 0$
3) Calculate E_{ss} for ramp input with slope A when $K_2 = 0$
4) Calculate E_{ss} for ramp input with slope A when $K_2 \neq 0$
5) Given $K_1=1.2$, $K_2=8.4$, and $T=0.5$, what value of K gives $K_v=6$ for unit ramp input. Find the corresponding steady-state error. Sketch the input and output as functions of time in that case.



Exercise (2)

$$G(s) = \frac{K(K_1 S + K_2)}{S(1 + TS)}$$

1) When $K_2 = 0$, the above G(s) is reduced to:

$$G(s) = \frac{KK_1}{1 + TS}$$

$$K_p = \lim_{S \to 0} G(S) = KK_1$$

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + KK_1}$$

For a step input with magnitude A;

$$E_{ss} = \frac{A}{1 + KK_1}$$

2) When $K_2 \neq 0$, the open-loop T.F reverts to the original form:

$$G(s) = \frac{K(K_1 S + K_2)}{S(1 + TS)}$$

Exercise (2)

Which represent a type 1 system.

$$K_p = \lim_{S\to 0} G(S) = \infty$$

$$E_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

3) When $K_2 = 0$ and the input is ramp;

$$K_v = \lim_{S\to 0} S\,G(S) = 0$$

$$e_{ss} = \frac{A}{K_v} = \infty$$

When $K_2 \neq 0$,

$$K_v = \lim_{S\to 0} S\,G(S) = KK_2$$

$$e_{ss}(t) = \frac{A}{K_v} = \frac{A}{KK_2}$$

4. Given $K_1 = 1.2$, $K_2 = 8.4$, and $T = 0.5$, it is required that

$$K_v = 6 = 8.4K$$

Hence the result

$$K = 0.714$$

The steady-state error of the closed-loop system becomes

$$e_{ss}(t) = \frac{1}{K_v} = 0.167$$

Exercise (3)

For the control system shown below in Fig A,

a) Determine the values of gain K and the time constant T so that the system response for unit-step input is as shown in Fig. B.

b) With these values of K and T, obtain in part (a), find the rise time and peak time and percentage overshoot.

c) Calculate the position error coefficient and the steady-state error.

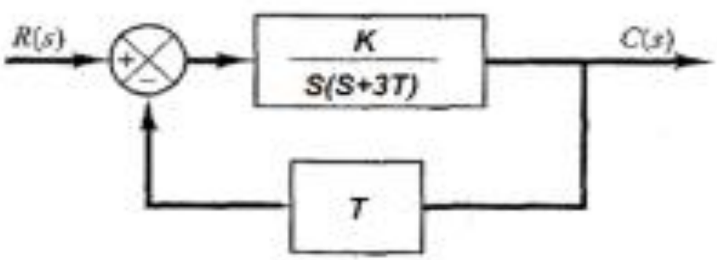


Fig. A

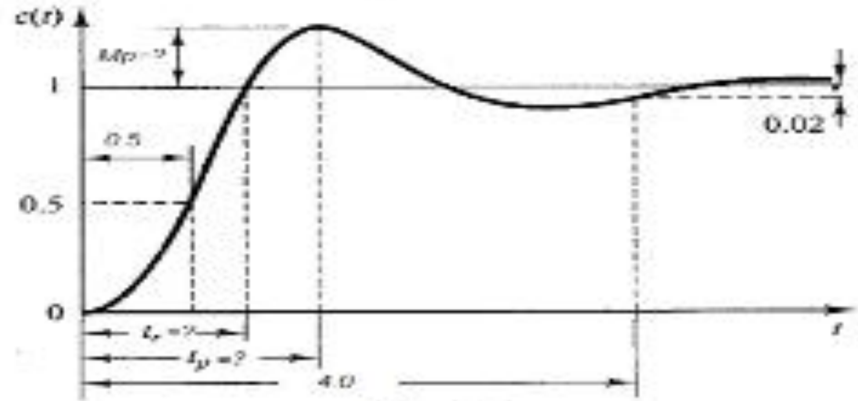


Fig. B

From system response (Fig. B),

$T_S = 4$ (based on 2% tolerance) and $T_d = 0.5$

since

$$T_s = \frac{4}{\xi \, \omega_n} = 4$$

Therefore,

Exercise (3)

$$\xi\,\omega_n = 1$$

Also, we have the delay time $T_d = 0.5$

$$T_d = \frac{1 + 0.7\xi}{\omega_n} = 0.5$$

Multiply both sides by ξ, then

$$T_d = \frac{\xi + 0.7\xi^2}{\xi\,\omega_n} = 0.5$$

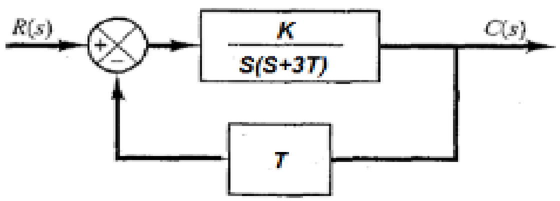
But we get that, $\xi\,\omega_n = 1$, therefore

$$0.7\xi^2 + \xi - 0.5 = 0$$

Solving this equation

$$\xi = 0.392281\ (accepted)$$
$$\xi = -1.82085\ (rejected)$$
$$\omega_n = \frac{1}{\xi} = 2.5492\ rad/s$$

From the system block diagram



The system T.F.

$$\frac{C(s)}{R(s)} = \frac{K}{S^2 + 3Ts + KT}$$

The system characteristic equation is

$$S^2 + 3Ts + KT = 0$$

The standard form of 2nd order system characteristic equation is

Exercise (3)

The system characteristic equation is

$$S^2 + 3Ts + KT = 0$$

The standard form of 2nd order system characteristic equation is

$$S^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$$

By comparing

$$2\xi\omega_n = 3T$$

$$T = \frac{2}{3}$$

Also,

$$\omega_n{}^2 = KT$$

$$K = \frac{\omega_n{}^2}{T} = 9.7476$$

The % maximum over shoot M_p

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = 26.19\,\%$$

Rise Time:

$$T_r = \frac{\pi - \beta}{\omega_n\sqrt{1-\xi^2}}$$

B = $\cos^{-1}(\xi)$ = 66.90349° = 1.167686 rad

$$T_r = \frac{\pi - 1.167686}{2.5492\sqrt{1-0.392281^2}} = 0.842\ sec.$$

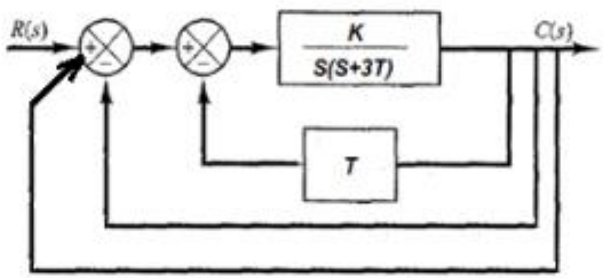
Peak Time:

Exercise (3)

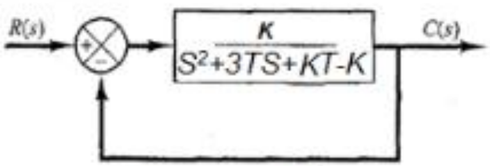
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$T_p = \frac{\pi}{2.5492\sqrt{1-0.392281^2}} = 1.34\ sec.$$

To get the steady-state error and position error coefficient, the system must be unity feedback, so we will add +ve and –ve feedback as shown in fig



Then the unity feedback system will be



$$K_p = \lim_{S\to 0} G(s) = \frac{K}{KT-K} = \frac{1}{T-1} = \frac{1}{0.6667-1} = -3$$

$$E_{ss} = \frac{1}{1+K_p} = \frac{1}{1-3} = -0.5$$

Exercise (1)
Lyapunov's second method for stability

Consider the following system

$$\dot{X} = cosx - x^3 + u$$

1) ***Suggest candidate Lyapunov function for that system.***
2) ***Suggest control command to stabilize that system at the origin.***
3) ***Define the type of stability.***
4) ***Discuss the "simplicity of the proposed control command". Can we simplify it?***

Exercise (1) solution
Lyapunov's second method for stability

$$\dot{X} = cosx - x^3 + u$$

Sol.: We assume the following energy function V(x):

$$V(x) = \frac{1}{2}x^2$$

1- $\boldsymbol{V(x) = 0}$ if and only if $\boldsymbol{x = 0}$;

2- $\boldsymbol{V(x) > 0}$ if and only if $\boldsymbol{x \neq 0}$;

Lyapunov candidate

Stable

3- $\boldsymbol{\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx} \cdot \frac{dx}{dt} = \frac{1}{2}2x.\dot{x} = x(cosx - x^3 + u)}$

Let u $= \boldsymbol{-cosx + x^3 - x}$ $\Rightarrow$ $\dot{V}(x) = -x^2 \leq 0$

Lyapunov function

Exercise (1) solution
Lyapunov's second method for stability

Sol.: to define the type of stability, we have to perform two tests as follow:

4- $\dot{\boldsymbol{V}}(\boldsymbol{x}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{V}(x) = \frac{\boldsymbol{dV}(\boldsymbol{x})}{\boldsymbol{dx}} \cdot \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -x^2 = \mathbf{0}$ if and only if x = 0; AS

5- $\lim_{\|\boldsymbol{x}\| \to +\infty} \dot{\boldsymbol{V}}(\boldsymbol{x}) = \lim_{\|\boldsymbol{x}\| \to +\infty} -x^2 = -\infty$; GS

Global Asymptotically Stable (GAS)

The system is globally asymptotically (GAS) stable in sense of Lyapunov

Exercise (1) solution with another (u)
Lyapunov's second method for stability

$$\dot{X} = cosx - x^3 + u$$

Sol.: We assume the following energy function V(x):

$$V(x) = \frac{1}{2}x^2$$

1- $\boldsymbol{V(x) = 0}$ if and only if $\boldsymbol{x = 0}$;

2- $\boldsymbol{V(x) > 0}$ if and only if $\boldsymbol{x \neq 0}$;

Lyapunov candidate

3- $\boldsymbol{\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx} . \frac{dx}{dt} = \frac{1}{2}2x . \dot{x} = x(cosx - x^3 + u)}$

Stable

Lyapunov function

Let u $= -\boldsymbol{cosx}$ $\Rightarrow$ $\dot{V}(x) = -x^4 \leq 0$

Exercise (1) solution "continue" Lyapunov's second method for stability

Sol.: to define the type of stability, we have to perform two tests as follow:

4- $\dot{\boldsymbol{V}}(\boldsymbol{x}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{V}(x) = \frac{\boldsymbol{dV}(\boldsymbol{x})}{\boldsymbol{dx}} \cdot \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -x^4 = \boldsymbol{0}$ if and only if x = 0; AS

5- $\lim_{\|\boldsymbol{x}\| \to +\infty} \dot{\boldsymbol{V}}(\boldsymbol{x}) = \lim_{\|\boldsymbol{x}\| \to +\infty} -x^4 = -\infty$; GS

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Exercise (1) solution with another (u)
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Lyapunov candidate

3- $\boldsymbol{\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx}.\frac{dx}{dt} = \frac{1}{2}2x.\dot{x} = x(cosx - x^3 + u)}$

Stable

Let u $= \boldsymbol{-cosx + x^3 - kx}$ $\Rightarrow$ $\dot{V}(x) = -kx^2 \leq 0$ with k > 0

Lyapunov function

Exercise (1) solution "continue"
Lyapunov's second method for stability

Sol.: to define the type of stability, we have to perform two tests as follow:

4- $\dot{\boldsymbol{V}}(\boldsymbol{x}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{V}(x) = \frac{\boldsymbol{dV(x)}}{\boldsymbol{dx}} \cdot \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -kx^2 = \mathbf{0}$ if and only if x = 0 — AS

5- $\lim_{\|\boldsymbol{x}\| \to +\infty} \dot{\boldsymbol{V}}(\boldsymbol{x}) = \lim_{\|\boldsymbol{x}\| \to +\infty} -kx^2 = -\infty$; — GS

Global Asymptotically Stable (GAS)

The system is globally asymptotically (GAS) stable in sense of Lyapunov

Exercise (2)
Lyapunov's second method for stability

Consider the following system

$$\dot{x}_1 = -x_1 + x_3^2$$
$$\dot{x}_2 = -x_2$$
$$\dot{x}_3 = x_3 - x_3^2 + u$$

1) ***Suggest candidate Lyapunov function for that system.***
2) ***Suggest control command to stabilize that system at the origin.***
3) ***Define the type of stability.***

Exercise (1) solution
Lyapunov's second method for stability

Sol.: We assume the following energy function V(x):

$$V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2$$

1- $\boldsymbol{V(x) = 0}$ if and only if $\boldsymbol{x = 0}$;

2- $\boldsymbol{V(x) > 0}$ if and only if $\boldsymbol{x \neq 0}$;

Lyapunov candidate

3- $\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx} \cdot \frac{dx}{dt} = \frac{1}{2}2x_1.\dot{x}_1 + \frac{1}{2}2x_2.\dot{x}_2 + \frac{1}{2}2x_3.\dot{x}_3$

$$= x_1.(-x_1 + x_3^2) + x_2.(-x_2) + x_3.(x_3 - x_1^3 + u)$$

$$= -x_1^2 - x_2^2 + x_3\,(x_3 + x_1x_3 - x_1^3 + u)$$

Stable

Let $u = -x_3(1 + x_1) + x_1^3 - x_3 \quad \Rightarrow \quad \dot{V}(x) = -x_1^2 - x_2^2 - x_3^2 \leq 0$

Lyapunov function

Exercise (1) solution "continue"
Lyapunov's second method for stability

Sol.: to define the type of stability, we have to perform two tests as follow:

4- $\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx}.\frac{dx}{dt} = -x_1^2 - x_2^2 - x_3^2 = 0$ if and only if x = 0; AS

5- $\lim_{\|x\|\to+\infty} \dot{V}(x) = \lim_{\|x\|\to+\infty} -x_1^2 - x_2^2 - x_3^2 = -\infty$; GS

Global Asymptotically Stable (GAS)

The system is globally asymptotically (GAS) stable in sense of Lyapunov

Exercise (3)
Lyapunov's second method for stability

Consider the following system

$$\dot{x} = \sqrt{x} + \sin^2 x + u$$

1) ***Prove that*** $V(\boldsymbol{x}) = 1 - \cos x$ ***is candidate Lyapunov function.***
2) ***Suggest control command to stabilize that system at the origin.***
3) ***Define the type of stability.***

Exercise (3) solution
Lyapunov's second method for stability

Sol.:

1- For the given energy function V(x):

$$\boldsymbol{V}(\boldsymbol{x}) = 1 - \cos x$$

1- $\boldsymbol{V}(\boldsymbol{x}) = \mathbf{0}$ if and only if $\boldsymbol{x} = \mathbf{0}$;

2- $\boldsymbol{V}(\boldsymbol{x}) > \mathbf{0}$ if and only if $\boldsymbol{x} \neq \mathbf{0}$;

Lyapunov candidate

2- We assume the following energy function V(x):

$$\mathrm{V}(x) = \frac{1}{2}x^2$$

1- $\boldsymbol{V}(\boldsymbol{x}) = \mathbf{0}$ if and only if $\boldsymbol{x} = \mathbf{0}$;

2- $\boldsymbol{V}(\boldsymbol{x}) > \mathbf{0}$ if and only if $\boldsymbol{x} \neq \mathbf{0}$;

Lyapunov candidate

Exercise (3) solution "continue"
Lyapunov's second method for stability

Sol.: to define the type of stability, we have to perform two tests as follow:

3- $\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx}.\frac{dx}{dt} = x.\dot{x} = x(\sqrt{x} + \sin^2 x + u)$

Let $u = -\sqrt{x} - \sin^2 x - x \Rightarrow \qquad \dot{V}(x) = -x^2 \le 0$

Lyapunov function

Stable

4- $\dot{V}(x) = \frac{d}{dt}V(x) = \frac{dV(x)}{dx}.\frac{dx}{dt} = -x^2 = 0$ if and only if x = 0;

AS

5- $\lim_{\|x\| \to +\infty} \dot{V}(x) = \lim_{\|x\| \to +\infty} -x^2 = -\infty???$ (no because $\sqrt{\infty}$ *is undefined*) ;

Not GS

The system is asymptotically (AS) stable in sense of Lyapunov

Exercise (4)
Lyapunov's second method for stability

Consider the following system

$$\dot{x}_1 = sinx_1 - x_1^3 + x_2$$
$$\dot{x}_2 = -x_1 + u$$

1) ***Suggest candidate Lyapunov function for that system.***
2) ***Suggest control command to stabilize that system at the origin.***
3) ***Define the type of stability.***